

Modes of Atomic Motions in Liquid Helium by Inelastic Scattering of Neutrons

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INFORMATION concerning the atomic arrangements and modes of atomic motions in liquids and solids may be determined from measurements of the scattering of neutrons or x rays. Such determinations are particularly important in the case of liquid helium because at low temperatures the liquid is nearly in its ground state and the details of the λ transition have not been explained. The measured small-angle x-ray¹ and neutron scattering² is almost in agreement with that predicted theoretically. Long-wavelength neutrons^{2,3} have been used in an unsuccessful search for extra scattering⁴ expected to result from the effect of the Bose-Einstein condensation in the He-II region of the transition. Both neutrons⁵⁻⁷ and x rays^{8,9} have been

This paper reports a remeasurement and an extension of the existing measurements of the wavelength change of neutrons inelastically scattered from liquid helium to higher precision and statistical accuracy. For these measurements, a new high-resolution double-crystal spectrometer at the Chalk River N.R.U. reactor was used. Analysis of the dispersion curve shows that at

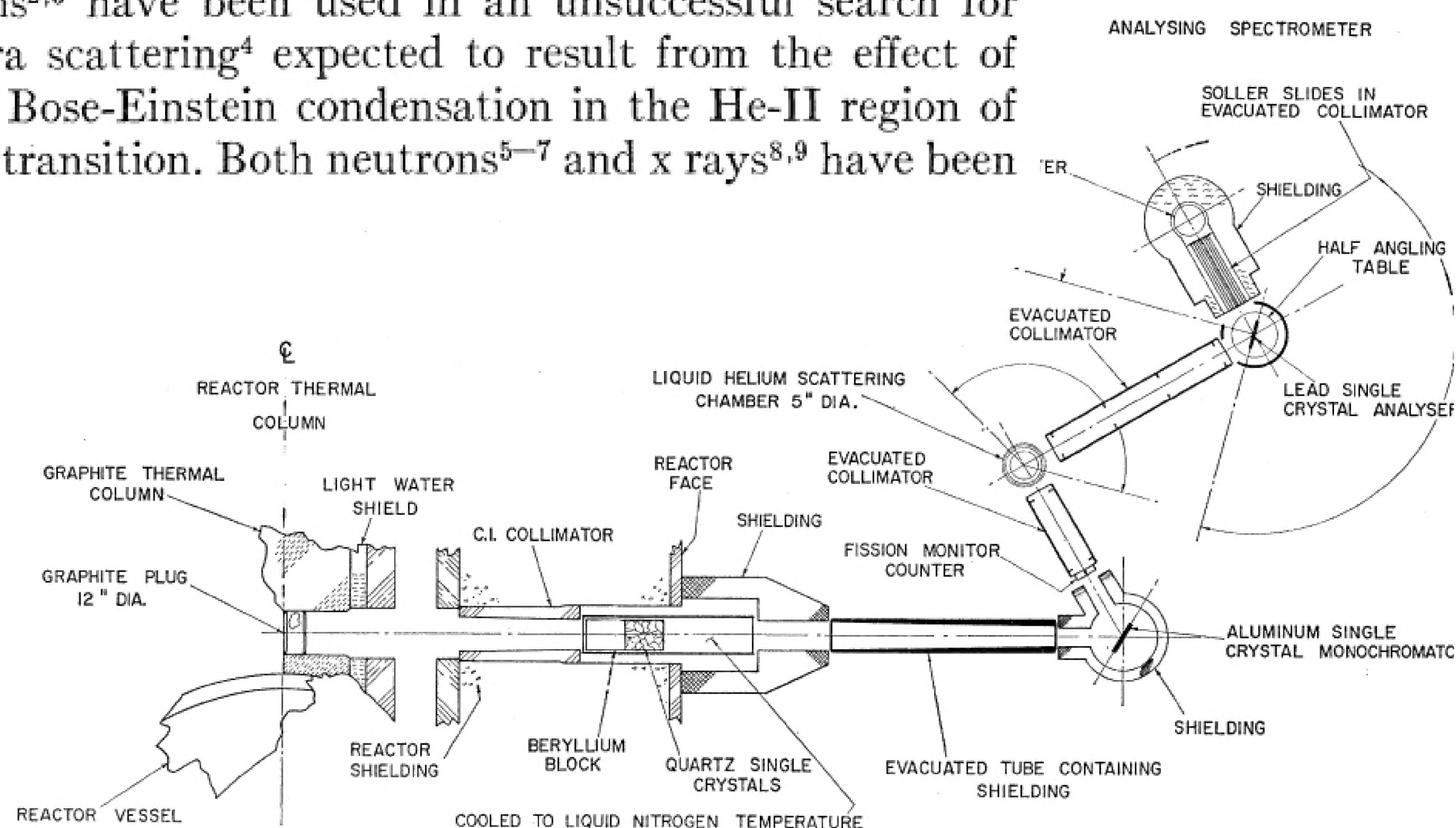
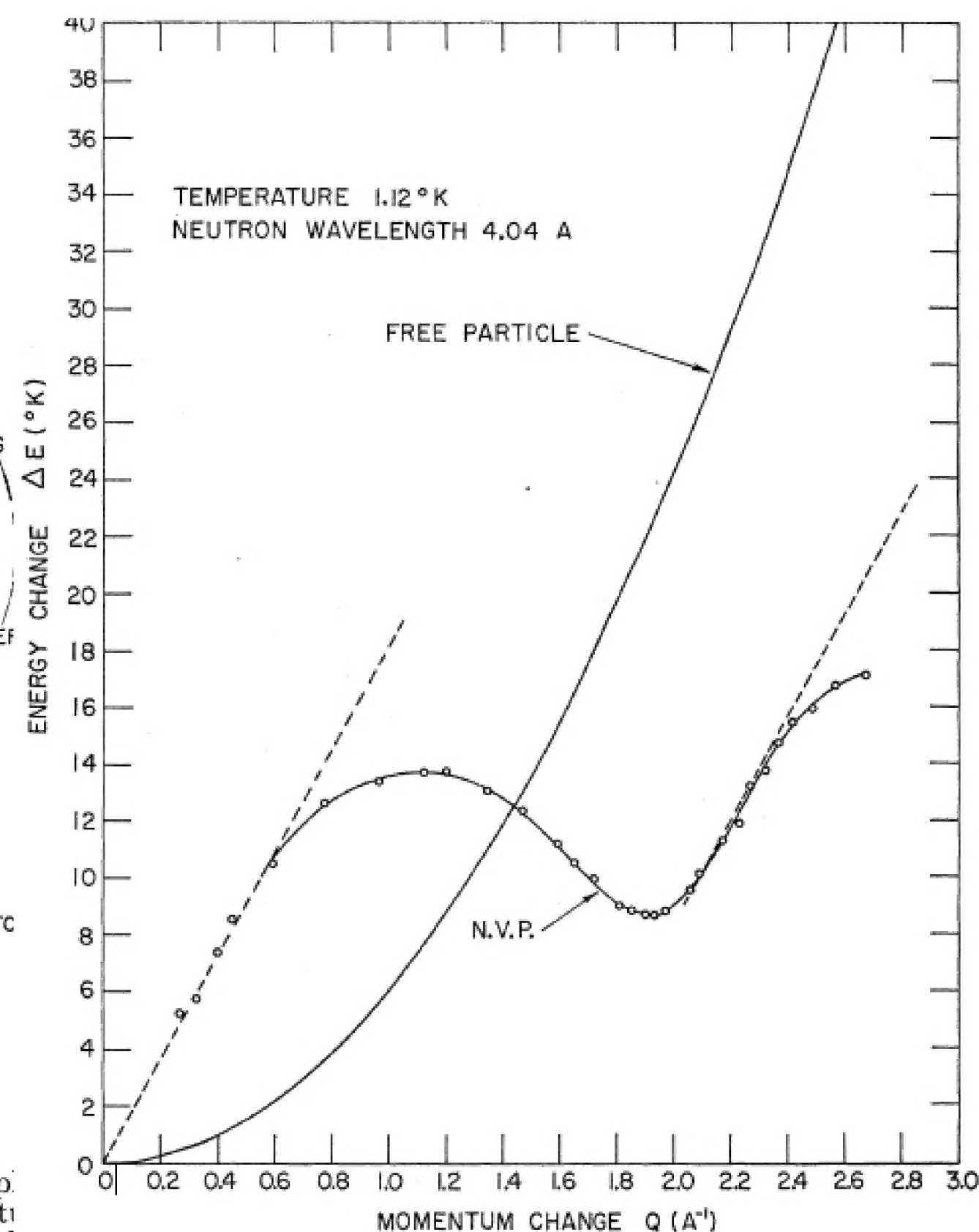


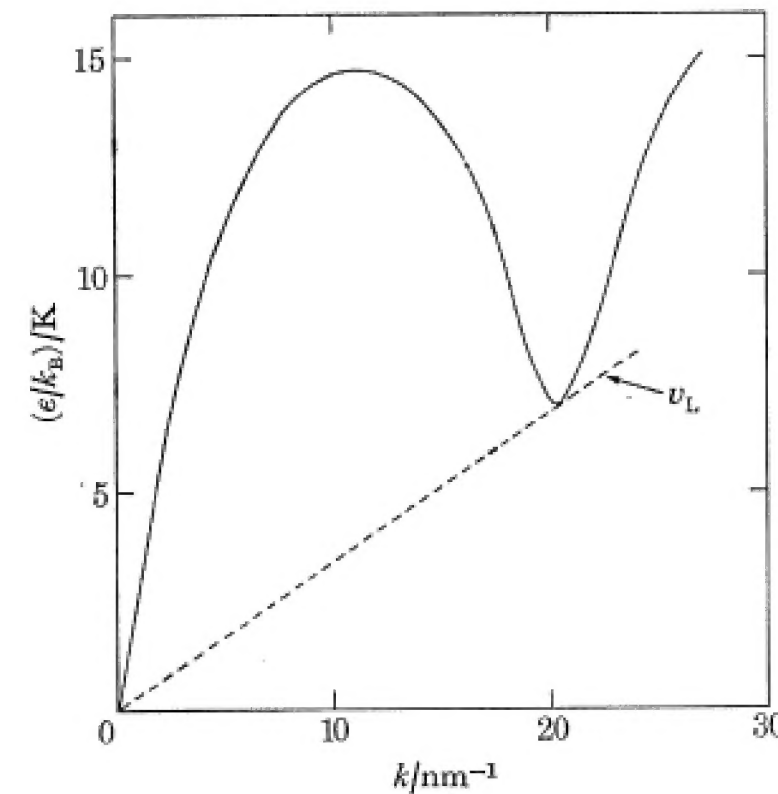
FIG. 1. The Chalk River liquid helium double-crystal neutron spectrometer (not to scale). Neutrons from the 12 in. diameter gap plug at the center line of the N.R.U. thermal column, after transmission through beryllium and quartz (at liquid nitrogen temperature).



yielding

$$\frac{d\epsilon}{dk} = \epsilon/k.$$

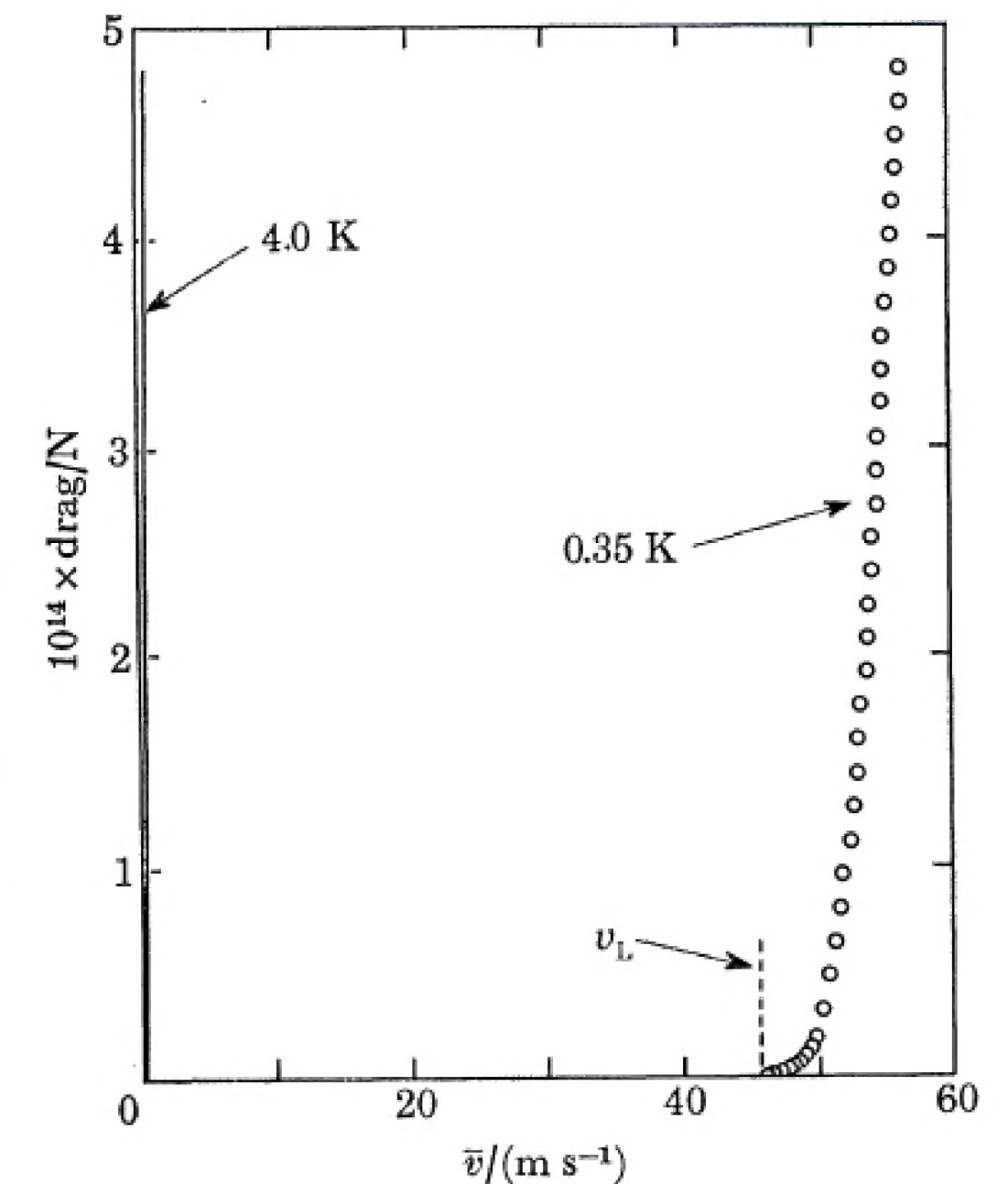
Substituting for ϵ from (5) we can then determine the value of k with and thence, by using (4) and (5), we can determine v_L precisely. For a simpler approach suffices. We note that (6) implies that v_L is specified by the gradient of a line



drawn from the origin to make a tangent with the roton region of the curve, contacting it very close to the minimum at (Δ, k_0) , as indicated in figure 2. In fact

$$v_L \simeq \Delta/\hbar k_0 \quad (7)$$

within 1 %. Inserting experimental values of Δ and k_0 (Donnelly 1972), we find that $v_L = 58 \text{ m s}^{-1}$ at s.v.p., and $v_L = 46 \text{ m s}^{-1}$ close to the solidification pressure at $25 \times 10^5 \text{ Pa}$.



The other possible experimental approach is, of course, that depicted in figure 1: to move an object through stationary superfluid. Negative and positive ions constitute particularly convenient objects for this purpose. They can readily be injected into the liquid by means of a variety of different techniques, they can be moved through the liquid by application of electric fields, and their arrival at an electrode can be observed as a pulse of current. The so-called ions which can exist in liquid helium are, in fact semi-macroscopic objects with radii of *ca.* 1 nm and effective masses of *ca.* $100m_4$, where m_4 is the mass of a ^4He atom. Numerous investigations of ion motion in liquid ^4He have been carried out, and have been reviewed, with extensive bibliographies, by

The Breakdown of Superfluidity in Liquid ^4He : An Experimental Test of Landau's Theory

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On the Bose-Einstein Condensation

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IN his well-known papers¹ on the degeneracy of an ideal gas, Einstein mentioned a peculiar condensation phenomenon of the ideal "Bose-Einstein" gas. This very interesting discovery, however, has not appeared in the textbooks, probably because Uhlenbeck in his thesis² questioned the correctness of Einstein's argument. Since, from the very first, the mechanism appeared to be devoid of any practical significance, all real gases being condensed at the temperature in question, the matter has never been examined in detail; and it has been generally supposed that there is no such condensation phenomenon.

In discussing some properties of liquid helium, I recently realized that Einstein's statement has been erroneously discredited; moreover, some support could be given to the idea that the peculiar phase transition (" λ -point"), that liquid helium undergoes at 2.19°K, very probably has to be regarded as the condensation phenomenon of the Bose-Einstein statistics, distorted, of course, by the presence of molecular forces and by the fact that it manifests itself in the liquid and not in the gaseous state. In a preliminary note,³ the

ary molecules of a degenerate Bose-Einstein gas which we have denoted by N_0 and which indeed may be considered as belonging to a particular phase, do not, of course, disappear mysteriously from space; they do contribute to the density as any other molecules. They do *not contribute, however, to the pressure*, since their kinetic energy (and momentum), is zero. If one likes analogies, one may say that there is actually a *condensation, but only in momentum space*, and not in ordinary space, i.e., an equilibrium of two phases, *one* containing the molecules N_0 of momentum zero and occupying in the space of momenta, a zero volume; and *another one* showing a distribution over all momenta similar to that which is realized for $T > T_0$. In ordinary space, however, no separation of phases is to be noticed.

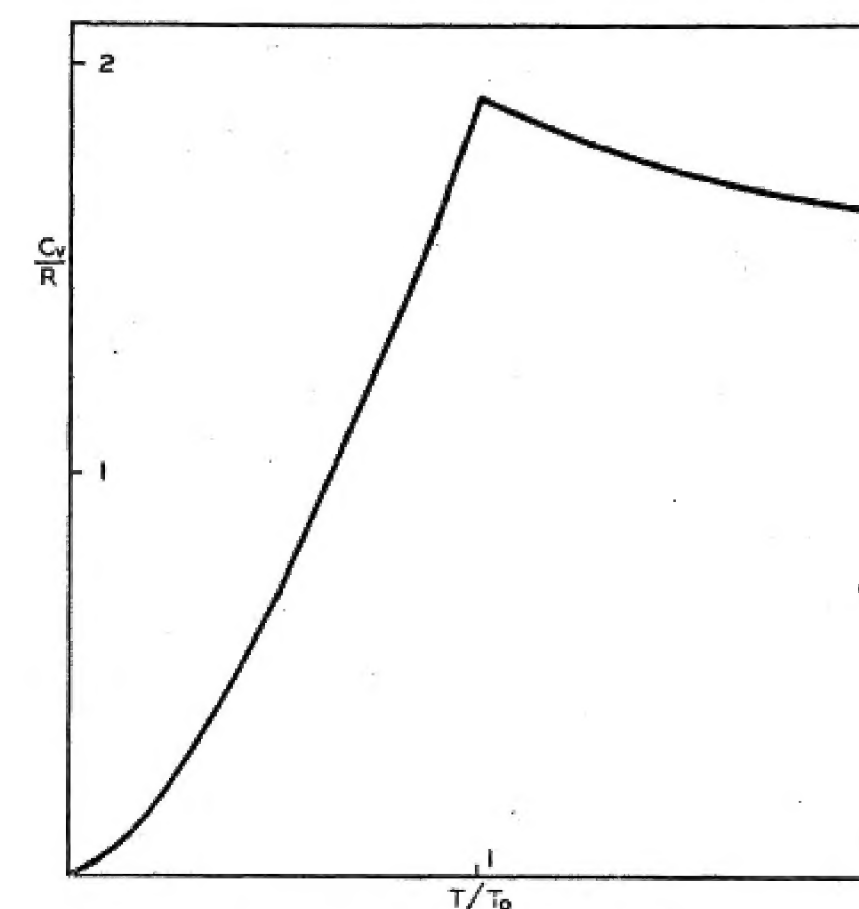


FIG. 4. Specific heat of an ideal Bose-Einstein gas.

The λ -Phenomenon of Liquid Helium and the Bose-Einstein Degeneracy

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(3) In his well-known papers, Einstein has already discussed a peculiar condensation phenomenon of the 'Bose-Einstein' gas; but in the course of time the degeneracy of the Bose-Einstein gas has rather got the reputation of having only a purely imaginary existence. Thus it is perhaps not generally known that this condensation phenomenon actually represents a discontinuity of the derivative of the specific heat (phase transition of third order). In the accompanying figure the specific heat (C_v) of an *ideal* Bose-Einstein gas is represented as a function of T/T_0 where

$$T_0 = \frac{h^2}{2\pi m^* k} \left(\frac{n}{2,615} \right)^{2/3}$$

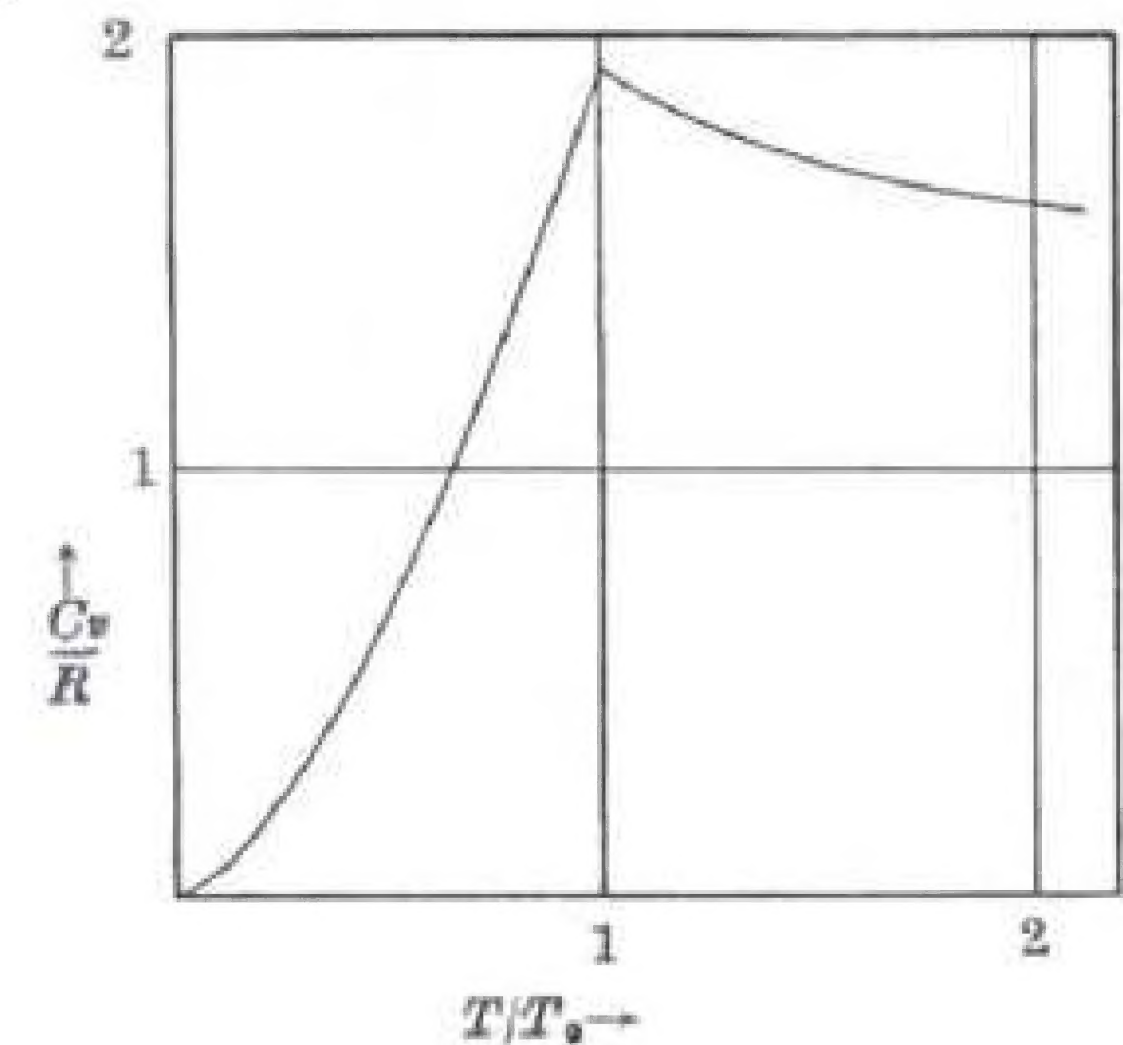
With m^* = the mass of a He atom and with the mol. volume $N/n = 27.6 \text{ cm}^3$ one obtains $T_0 = 3.09^\circ$. For $T \leq T_0$ the specific heat is given by

$$C_v = 1.92 R (T/T_0)^{3/2}$$

and for $T \geq T_0$ by

$$C_v = \frac{3}{2} R \left[1 + 0.231 \left(\frac{T_0}{T} \right)^{3/2} + 0.046 \left(\frac{T_0}{T} \right)^3 + \dots \right]$$

The entropy at the transition point T_0 amounts to $1.28 R$ independently of T_0 .



(4) Though actually the λ -point of helium resembles rather a phase transition of second order, it seems difficult not to imagine a connexion with the condensation phenomenon of the Bose-Einstein statistics. The experimental values of the temperature of the λ -point (2.19°) and of its entropy ($\sim 0.8 R$) seem to be in favour of this conception. On the other hand, it is obvious that a model which is so far away from reality that it simplifies liquid helium to an ideal gas, cannot, for high temperatures, yield but the value $C_v = 3/2 R$, and also for low temperatures the ideal Bose-Einstein gas must, of course, give too great a specific heat, since it does not account for the gradual 'freezing in' of the Debye frequencies.

Condensate and final-state effects in superfluid ^4He

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Bose-Einstein condensation plays a special role in quantum physics. Bose¹ and Einstein² proposed that a gas of non-interacting bosons would condense into a state characterized by macroscopic occupation of one single-particle quantum state. London³ suggested that this condensation is the origin of superfluidity in liquid ^4He at temperatures below $T_\lambda = 2.17$ K. Superconductivity in a wide spectrum of materials is now identified with condensation of paired fermions (composite bosons).⁴ Spectacular Bose condensation in which nearly the entire system is condensed into a single atomic state has recently been observed in trapped, dilute gases of alkali atoms.^{5,6}

Liquid ^4He remains the most accessible pure Bose liquid in nature and a key model system.⁷⁻⁹ Because of the strong interaction between the ^4He atoms in the liquid, particularly the repulsive core, the fraction n_0 of atoms that condense into the zero-momentum ($\mathbf{k}=0$) state is small, approximately 10% at $T=0$.¹⁰⁻¹⁶ The clear verification that there is indeed a condensate and the accurate determination of n_0 remain an important goal. Taking advantage of significant improvements in neutron source intensity and instrumentation, we present accurate data and analysis methods which we believe show unambiguously that superfluid ^4He has a condensate, with a condensate fraction $n_0 = 6.0 \pm 2.0\%$ at $T=1.6$ K.

The measurements were carried out at the ISIS pulsed spallation neutron source at the Rutherford Appleton Laboratory in the United Kingdom. The instrument used was the high-energy direct geometry chopper spectrometer MARI with incident energies up to 1000 meV possible. More than 650 ^3He gas detectors provide an almost continuous coverage of scattering angles, ϕ , between 3° and 135° in steps of 0.43° . This makes it possible to measure a large range in momentum and energy transfer in a single experiment scan. Due to the pulsed nature of the source, data collection is performed in time-of-flight (TOF) in which the time of arrival of a neutron in the detector, measured from when they leave the moderator, determines its energy loss or gain after scattering from the sample. The momentum transfer depends

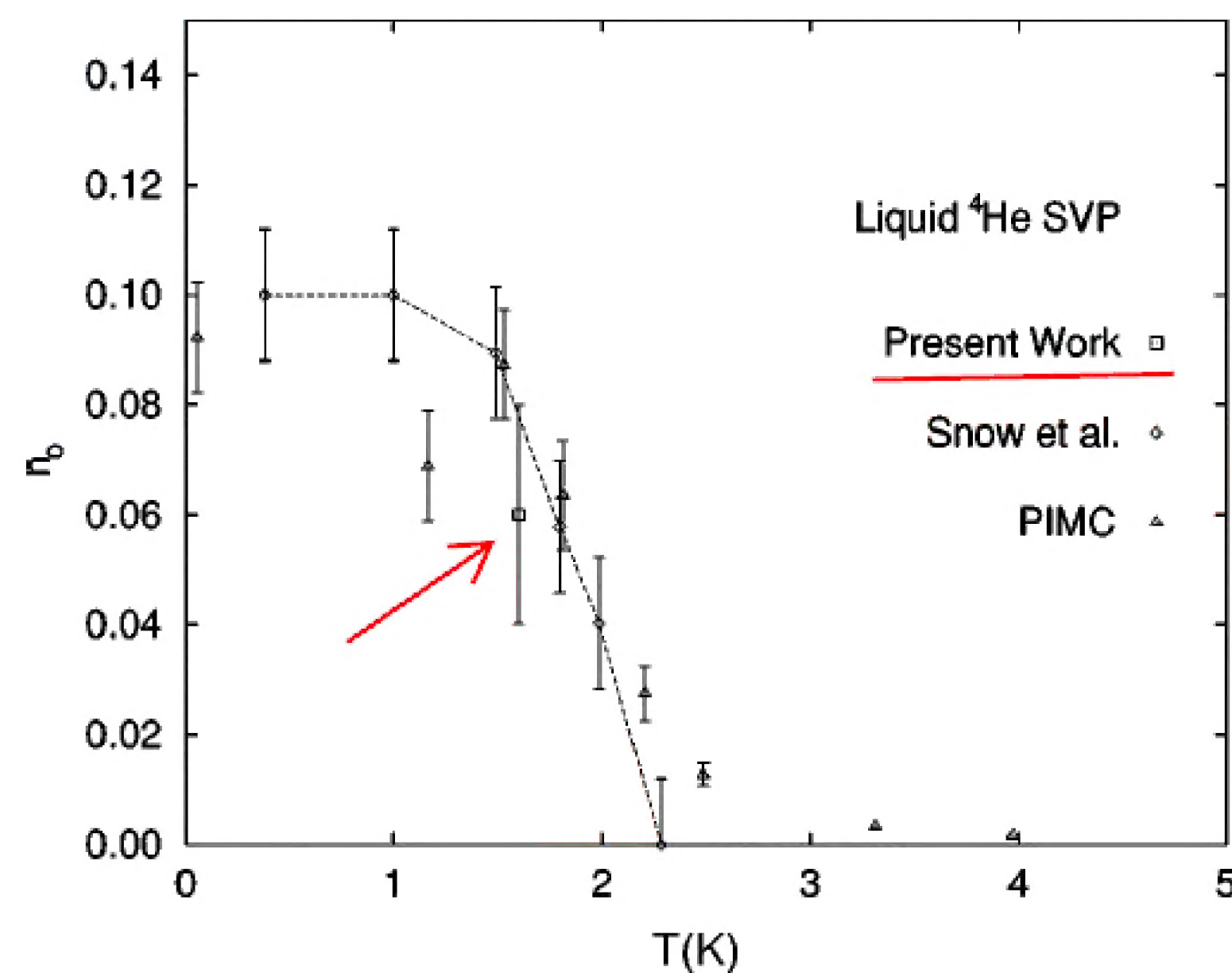


FIG. 8. Condensate fraction in liquid ^4He at SVP. Triangles are